Roll No. **Total No. of Pages :02**

Total No. of Questions :18

B.Tech. (Bio Technology/Civil Engineering/Computer Science & Engineering/Electrical & Electronics Engineering/Electrical Engineering/Electronics & Communication Engineering/Information Technology/Mechanical Engineering)(Sem.–1)

ENGINEERING MATHEMATICS-I

Subject Code :BTAM-101

M.Code :54091

Date of Examination : 01-07-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION - B &C. have FOUR questions each.**
3. **Attempt any FIVE questions from SECTION B& C carrying EIGHT marks each.**
4. **Select atleast TWO questions from SECTION - B &C.**

SECTION-A

Solve the following:

1. Find the percentage error in the area of an ellipse when an error of +1 percent is made in measuring the major and minor axes.
2. If $x = r\cos\theta$ and $y = r\sin\theta$, Verify that $\frac{\partial(x,y)}{\partial(r,\theta)} \times \frac{\partial(r,\theta)}{\partial(x,y)} = 1$.
3. Find the radius of the curvature of $y^2 = 4ax$ at any point (x, y).
4. State Greens theorem in the plane.
5. Find the equation of tangent plane for the surface $xyz = 6$ at (1, 2, 3).
6. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$.
7. State Stoke's theorem.
8. Find the gradient of the function $\phi = y^2 - 4xy$ at (1,2).
9. Show that the vector field given by $\vec{F} = (-x^2 + yz)\hat{i} + (4y - z^2x)\hat{j} + (2xz - 4z)\hat{k}$ is solenoidal.
10. Define homogenous function.

SECTION-B

11. Use Lagrange's method to find the minimum value of $x^2 + y^2 + z^2$ subject to the conditions $x + y + z = 1$ and $xyz + 1 = 0$.

12. If $U = \tan^{-1} \dfrac{x^3 + y^3}{x - y}$.

    Prove that $x^2 \dfrac{\partial^2 U}{\partial x^2} + 2xy \dfrac{\partial^2 U}{\partial x \partial y} + y^2 \dfrac{\partial^2 U}{\partial x^2} = \sin 4u - \sin 2u = 2 \cos 3u$.

13. a) Find all the asymptotes of the curve

    $y^3 - 3x^2y + xy^2 - 3x^3 + 2y^2 + 2xy + 4x + 5y + 6 = 0$.

    b) Find the moment of inertia of the area between $y = sinx$ from $x = 0$ to $x = n$ and x–axis about each axis.

14. Trace the curve $y^2 = \dfrac{x^3}{2a - x}$.

SECTION-C

15. a) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

    b) Evaluate $\iiint x^2 yz\,dxdydz$ over the region bounded by $x = 0, y = 0, z = 0, x + y + z = 1$.

16. Verify Gauss Divergence theorem for $\vec{F} = (x + y^2)\hat{i} - 2x\hat{j} + 2yz\hat{k}$ takenover tetrahedron bounded by coordinate planes and the plane $2x + y + 2z = 6$.

17. Prove that:

    a) $curl(\phi \vec{A}) = (grad\, \phi) \times \vec{A} + \phi\, curl\, \vec{A}$

    b) $\nabla^2 f(r) = f''(r) + \dfrac{2}{r} f'(r)$.

18. Verify Stoke's theorem for $\vec{F} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xz + z^2)\hat{k}$ over the surface of hemisphere $x^2+y^2+z^2 = 16$ above XOY plane.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.

B Tech – Sem 1

Computer Science & Engineering

(2022).

BTAM-101

54091

Section - A

Q1. find % error in area of ellipse, when error of +1 % is made on measuring the major and minor axis

area of ellipse $= \pi ab$.

$\log A = \log\pi + \log a + \log b$.

$\partial(\log A) = \partial(\log \pi) + \partial(\log a) + \partial(\log b)$.

$\frac{\partial A}{A} = 0 + \frac{\partial a}{a} + \frac{\partial b}{b}$.

$\frac{100}{A}\partial A = \frac{100}{a}\partial a + \frac{100}{b}\partial b$.

$\left[\text{Since } \frac{100\partial a}{a} = 1 \quad \frac{100}{b}\partial b = 1\right]$.

$\frac{100}{A}\partial A = 1+1=2$.

% error $= 2\%$.

Q2 $\quad x = r\cos\theta$

$y = r\sin\theta$

$$\longrightarrow \frac{\partial(x,y)}{\partial(r,\theta)} \times \frac{\partial(r,\theta)}{\partial(x,y)} = 1.$$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$$= r\cos^2\theta + r\sin^2\theta$$

$$= r.$$

$x^2 = r^2\cos^2\theta$

$y^2 = r^2\sin^2\theta$.

$\therefore r = \sqrt{x^2+y^2}$

$\tan\theta = \frac{y}{x}$.

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$$\frac{\partial(r,\theta)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} \\ \frac{-y}{x^2+y^2} & \frac{x}{x^2+y^2} \end{vmatrix}$$

$$= \frac{x^2}{(x^2+y^2)^{3/2}} + \frac{y^2}{(x^2+y^2)^{3/2}}$$

$$= (x^2+y^2)^{1-\frac{3}{2}} = \frac{1}{(x^2+y^2)^{1/2}}$$

$\therefore \quad \lambda \times \frac{1}{\lambda} = 1 = 1.$

proved ...

Q3. Radii of curvature $\boxed{y^2 = 4ax}$ at pt (x, y).

$$y^2 = 4ax$$
$$2y\frac{dy}{dx} = 4a.$$
$$y\frac{dy}{dx} = 2a.$$
$$\frac{dy}{dx} = \left(\frac{2a}{y}\right).$$
$$\frac{d^2y}{dx^2} = -\frac{2a}{y^2}\left(\frac{dy}{dx}\right).$$
$$= -\frac{2a}{y^2} \times \frac{2a}{y}$$
$$\frac{d^2y}{dx^2} = \frac{-4a^2}{y^3}$$

Rad of curvature.

$$R = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} \frac{d^2y}{dx^2}$$
$$= \left[1 + \frac{4a^2}{y^2}\right]^{3/2} \times -\frac{4a^2}{y^3}$$
$$= -(y^2 + 4a^2)^{3/2}\, 4a^2$$

Q4. $\boxed{\text{Green Theorem :-}}$

Let C be positively oriented, smooth and simple closed curve in a plane and D region bounded by C. If L and M are function of x and y (x, y) defined on open region, containing D and have continuous partial derivatives, then green Th stated as;

$$\oint_C M\,dx + N\,dy = \iint_D \left(\frac{\partial M}{\partial x} - \frac{\partial N}{\partial y}\right) dx\,dy.$$

Q5. Equation of tangent plane for surface $xyz = 6$ at $(1,2,3)$.

$$F(x\,y\,z) = xyz - 6 = 0.$$

pt of tangency $= (1,2,3)$.

$$F_x(x_0\,y_0\,z_0) + F_y(x_0\,y_0\,z_0) + F_z(x_0\,y_0\,z_0) = 0.$$

$\longrightarrow$

$\longrightarrow$ $yz(1,2,3)(x-1) + xz(1,2,3)(y-2) + xy(1,2,3)(z-3) = 0$

$\longrightarrow$ $2\times 3(x-1) + 3(y-2) + 2(z-3) = 0$

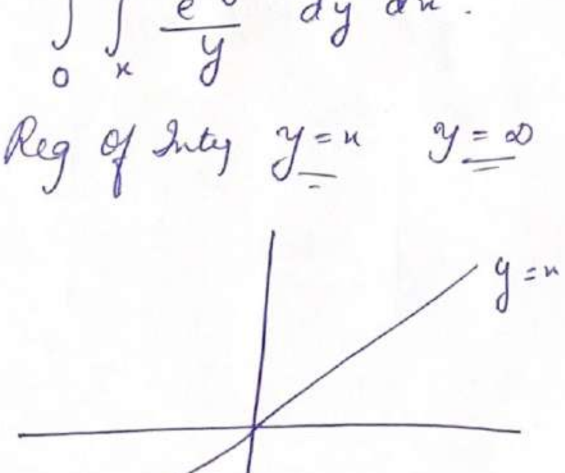
$\longrightarrow$ $6(x-1) + 3(y-2) + 2(z-3)$

$6x - 6 + 3y - 6 + 2z - 6$

$$\boxed{6x + 3y + 2z - 18 = 0}$$

Q6. $$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dy\,dx.$$

Reg of Integ $y = x$ $\quad y = \infty$



$$= \int_{y=0}^{\infty} \int_{x=0}^{y} \frac{e^{-y}}{y} \, dx \, dy.$$

$$= \int_{0}^{\infty} \frac{e^{-y}}{y} \left[x \right]_{0}^{y} dy.$$

$$= \int_{0}^{\infty} e^{-y} \, dy = 1.$$

Q7. Stokes Theorem :-

The surface integral of curl of a function over a surface bounded by a closed surface is equal to line integral of particular vector function around that surface

$$\oint_C \vec{F} \cdot \vec{dr} = \iint_S \underbrace{(\nabla \times \vec{F})}_{(\text{curl} F)} \cdot \vec{ds}.$$

Where C = closed curve.

S = surface bounded by C.

F = vector field whose comp have cont derivative in an open region of R^3 containing S.

Q8. gradient $\phi = (y^2 - 4xy)$ at $(1,2)$.

$\phi_x = -4y$

$\phi_y = 2y - 4x$.

$\nabla\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y}$.

$= \hat{i}(-4y) + \hat{j}(2y - 4x)$.

$= -4\hat{i} + \hat{j}(4-4)$.

$= -4\hat{i} + 0\hat{j}$.

Q9. $\vec{F}$ = solenoid (TP)

$\vec{F} = (x^2 + yz)\hat{i} + (4y - z^2x)\hat{j} + (2xz - 4z)\hat{k}$

is (solenoid).

Solenoidal $(\nabla \cdot \vec{F} = 0)$

$\hat{i}\frac{\partial F}{\partial x} + \hat{j}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z} = 0$.

$-2\cancel{x} + 4 + (2\cancel{x} - 4) = 0$

$\boxed{0 = 0}$ ✓

∴ field is solenoidal.

Q10. homogen function :-

A function is said to be homog of x and y if it can expressed in form of

$$x^n f\left(\frac{y}{x}\right) \text{ where } n = \text{degree of function}$$

eg $z = \frac{x^3 - y^3}{x+y} = \frac{x^3\left(1 - \frac{y^3}{x^3}\right)}{x(1 + y/x)}$

$= x^2 f(y/x)$.

Section-B :-

Q11. $F(xyz) = x^2 + y^2 + z^2 + \lambda(x+y+z-1) + \mu(xyz+1)$.

$\frac{\partial F}{\partial x} = 2x + \lambda(1) + \mu yz$ — 1.

$\frac{\partial F}{\partial y} = 2y + \lambda + \mu xz$. — 2.

$\frac{\partial F}{\partial z} = 2z + \lambda(\mu xy)$. — 3.

Subt 3 from 2.

$2(x-y) + \mu z(y-x) = 0$.

$x = y \mid \mu = 2/z$.

$y = z$ or $\mu = \frac{2}{x}$

$z = x$ or $\mu = \frac{2}{y}$.

$(x = y = z)$ or $\mu = \frac{2}{x} = \frac{2}{y} = \frac{2}{z}$.

Since $x + y + z = 1$

$3x = 1$

$x = 1/3 \therefore y = 1/3 \therefore z = 1/3$.

$\mu = 2/1/3 = 6$

$\therefore \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) = st\ pt$.

$$d^2F = F_{xx}(dx)^2 + F_{yy}(dy)^2 + F_{zz}(dz)^2 + 2F_{xy}\,dx\,dy + 2F_{yz}\,dy\,dz + 2F_{zx}\,dz\,dx.$$

$$= 2(dx)^2 + 2(dy)^2 + 2(dz)^2 + 2\times 6\times \frac{1}{3}dx\,dy + 2\times 6\times\frac{1}{3}dy\,dz + 2\times 6\times\frac{1}{3}dz\,dx.$$

$$= 2(dx+dy+dz)^2 > 0.$$

$\therefore F(xyz)$ is min at $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

and min Value $= \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2$

$= \frac{3}{9} = 1/3$.

Q12. $U = \tan^{-1}\dfrac{x^3+y^3}{x-y}$.

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u$$
$$= 2\cos 3u \sin u$$

$\longrightarrow$

$$\tan u = \frac{x^3+y^3}{x-y}$$

$$\tan u = x^2 f(y/x)$$

Euler Th:- $x\frac{\partial}{\partial x}(\tan u) + y\frac{\partial}{\partial y}(\tan u) = 2\tan u$.

$$x \sec^2 u\frac{\partial u}{\partial x} + y\sec^2 u\frac{\partial u}{\partial y} = 2\tan u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{2\tan u}{\sec^2 u}.$$

$$= 2\frac{\sin u}{\cos u}\times\cos^2 u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}. = \sin 2u.$$

diff w.r.t x.

$$x\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + y\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\times 0 = \cos 2u \times 2\frac{\partial u}{\partial x}.$$

$$x^2\frac{\partial^2 u}{\partial x^2} + xy\frac{\partial^2 u}{\partial y^2} = x(2\cos 2u - 1)\frac{\partial u}{\partial x} \quad \text{—}*$$

$\xrightarrow{x\times}$

Simly

$$\longrightarrow y^2\frac{\partial^2 u}{\partial y^2} + xy\frac{\partial^2 u}{\partial y^2} = y(2\cos 2u - 1)\frac{\partial u}{\partial y}. \quad \text{—}**$$

adding

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(\cos 2u-1)\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right)$$

$$=(\cos 2u-1)(\sin 2u)$$

$$=(\cancel{1}-2\sin^2 u-\cancel{1})(\sin 2u$$

$$\left\{x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=-2\sin^2 u\sin 2u\right\}$$

$$=-\sin 2u(+\sin 4u.$$

proved:-

Q13. Asymptotes of curve.

$$y^3-3x^2y+xy^2-3x^3+2y^2+2xy+4x+5y+6=0$$

$y=mx+c$ is asymptote

$$\phi_3=y^3-3x^2y+xy^2-3x^3$$

$$\phi_2=2y^2+2xy$$

$x=1 \quad y=m$

$$\phi_3(m)=m^3-3m+m^2-3$$

$$=m^3+m^2-3m-3$$

$$\phi_2=2y^2+2xy.$$

$$\phi_2(m)=2m^2+2m.$$

Since $\phi_3(m)=0.$

$$m^3+m^2-3m-3=0.$$

$m=-1$ $\quad (m+1)(m^2-3)=0$

$$m=-1,\ \pm\sqrt{3}$$

$$C = \frac{-\phi_2(m)}{\phi_3'(m)}.$$

$$\phi_3'(m) = 3m^2 + 2m - 3$$

$$\phi_2(m) = 2m^2 + 2m.$$

$\rightarrow \boxed{m = -1}$

$$C = \frac{-(2m^2 + 2m)}{3m^2 + 2m - 3}.$$

$$= \frac{-(2 \times 1 + 2 \times -1)}{3 - 2 - 3} = 0$$

$\rightarrow m = \sqrt{3}$

$$C = \frac{-(2m^2 + 2m)}{3m^2 + 2m - 3} = -1$$

$\rightarrow m = -\sqrt{3}$ $\quad C = \frac{-(2m^2 + 2m)}{3m^2 + 2m - 3} = -1$

$$y = mx + c$$

$m = -1$

$y = -x + 0$

$y = -x$

$m = \sqrt{3}$

$y = \pm\sqrt{3}x - 1$

$y = \sqrt{3}x - 1$

$m = -\sqrt{3}.$

$y = -\sqrt{3}x - 1.$

$y = -\sqrt{3}x - 1$

asymptote: asy of curve || to y axis by $2a-x=0$.

$\therefore x = 2a$.

Sp:

$$y = \frac{x^{3/2}}{\sqrt{2a-x}}$$

$$\frac{dy}{dx} = \frac{\sqrt{2a-x} \times \frac{3}{2}x^{1/2} - x^{3/2} \times \frac{1}{2\sqrt{2a-x}} \times -1}{(2a-x)}$$

$$\frac{dy}{dx} = \frac{\sqrt{x}\,(3a-x)}{(2a-x)^{3/2}}$$

$\rightarrow \frac{dy}{dx} = 0$

$$\frac{\sqrt{x}\,(3a-x)}{(2a-x)^{3/2}} = 0.$$

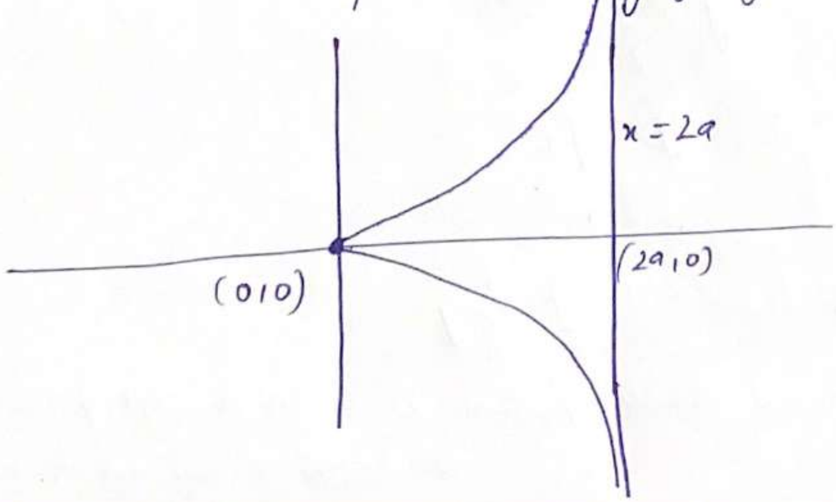
$x = 0 \quad x = 3a$.

Rej $(3a = x)$ because when $x = 3a$, y = Imag.

$x = 0$ & $y = 0$ tangent at $(0,0)$ parallel to xa

when $0 < x < 2a$ $\frac{dy}{dx} = +ve$

$\therefore$ positive value of y, $y \uparrow$ in $(0, 2a)$.



Q14.

$$y^2 = \frac{x^3}{(2a-x)}$$

$$\longrightarrow \boxed{y^2(2a-x) = x^3}$$

Symetry :- even power ∴ sy about x axis

Origin :- No const term, curve passes through Origin.

eq of tangent :- $2ay^2 - xy^2 - x^3 = 0$.

eq of tangent :- $2ay^2 = 0$

$y = 0$.

ie x axis ($y=0$) is tangent to curve at the origin.

pt of Int :- Put $y=0$ we get $x=0$

∴ curve meet x axis and y axis at Origin.

Region: $y^2 = \frac{x^3}{2a-x}$.

$$y = \frac{x\sqrt{x}}{\sqrt{2a-x}}$$

When $x < 0$, y = Imag.

∴ No curve portion is in lie to left of line $x=0$

∴ No portion of curve lie to right of line $x = 2a$

$\rightarrow$ $\boxed{\text{Section - C}}$

Q15. a. Volume Common to cy $(x^2+y^2=a^2)$ and $(x^2+z^2=a^2)$

(b.) Evaluate $\iiint x^2yz\,dx\,dy\,dz$ region bdd by curve $x=0$ $y=0$ $z=0$, $x+y+z=1$

$$= \int_0^1\int_0^{1-x}\int_0^{1-x-y} x^2yz \; dz\, dy\, dx.$$

$$= x^2y\left[\frac{z^2}{2}\right]_0^{1-x-y}.$$

$$= \int_0^1\int_0^{1-x} \frac{x^2y}{2}\left[(1-x-y)^2-0\right] dy\, dx.$$

$$\left[1-(x+y)\right]^2$$

$$= \int_0^1\int_0^{1-x} \frac{x^2}{2}y\left[1+(x+y)^2-2(x+y)\right].$$

$$= \int_0^1\int_0^{1-x} \frac{x^2y}{2}\left[1+x^2+y^2+2xy-2x-2y\right) dy\, dx.$$

$$= \int_0^1\int_0^{1-x} \frac{x^2y}{2}+\frac{x^4y}{2}+\frac{x^2y^3}{2}+\frac{\cancel{2}x^3y^2}{\cancel{2}}-\frac{\cancel{2}x^3y}{\cancel{2}}$$

$$-\frac{\cancel{2}x^2y^2}{\cancel{2}}\; dy\,dx$$

$$= \int_0^1\int_0^{1-x} \frac{x^2y}{2}+\frac{x^4y}{2}+\frac{x^2y^3}{2}\; dy\, dx.$$

$$= \frac{1}{24}\int_0^1 x^2(1-4x+6x^2-4x^3+x^4)\,dx.$$

$= \frac{1}{24}\left(\frac{x^3}{3} - x^4 + \frac{6x^5}{5} - \frac{2x^6}{3} + \frac{x^7}{7}\right]_0^1$

$= \frac{1}{2520}$ Ans.

$\underline{Q.17}$ Proove

(a.) $\text{Curl}(\phi\vec{A}) = (\text{grad}\,\phi)\times\vec{A} + \phi\,\text{curl}\,\vec{A}$.

$$\text{Curl}(\phi\vec{A}) = \hat{i}\times\frac{\partial}{\partial x}(\phi\vec{A}) + \hat{j}\times\frac{\partial}{\partial y}(\phi\vec{A}) + \hat{k}\times\frac{\partial}{\partial z}(\phi\vec{A}).$$

$$= \hat{i}\times\left(\frac{\partial\phi}{\partial x}\vec{A} + \phi\frac{\partial\vec{A}}{\partial x}\right) + \hat{j}\times\left(\frac{\partial}{\partial y}\phi\,\vec{A} + \phi\frac{\partial\vec{A}}{\partial y}\right) + \hat{k}\times\left(\frac{\partial\phi}{\partial z}\vec{A} + \phi\frac{\partial\vec{A}}{\partial z}\right).$$

$= \nabla\phi\times\vec{A} + \phi\,\text{curl}\,\vec{A}$.

$=$ RHS proved.

(b.) $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$.

$\boxed{\text{LHS}}$ $\nabla^2 f(r) = \underline{\underline{\nabla.\nabla f(r)}}$.

$= \nabla(f'\hat{r})$

$= \nabla\left(\frac{f'}{r}\vec{r}\right)$

$= \nabla\left(\frac{f'}{r}\right)\vec{r} + \frac{f'}{r}\,\text{div}\,\vec{r}$

$$= \left(\frac{r f'' - f'}{r^2}\right) \hat{r}\,\vec{r} + \frac{f'}{r} \cdot 3 \quad (\operatorname{div} \vec{r} = 3)$$

$$= r\left(\frac{f''}{r} - \frac{f'}{r^2}\right) \hat{r}\,\hat{r} + \frac{3u'}{r}$$

$$= r\left(\frac{f''}{r} - \frac{f'}{r^2}\right) + 3\frac{f'}{r} \cdot \left[\because \hat{r}\,\hat{r} = 1\right]$$

$$= f'' - \frac{f'}{r} + 3\frac{f'}{r} = f'' + \frac{2f'}{r}$$

(LHS = RHS)
proved.

Q18. $\vec{F} = (x^2+y-4)\hat{i} + 3xy\hat{j} + \hat{k}(2xz+z^2)$.

$S = x^2+y^2+z^2-16$.

$$\text{Curl } F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2+y & 3xy & 2xz+z^2 \end{vmatrix} = -2z\hat{j} + (3y+1)\hat{k}.$$

parametric Eq :-
$x = r\cos\theta \sin\phi$
$y = r\sin\theta \sin\phi$.
$z = r\cos\phi$.

put (z=0).

$$\int F \cdot dr = \int_0^{2\pi} F(r) \cdot r(\theta)\, d\theta$$

$$= \int_0^{2\pi} (16\cos^2\theta + 4\sin\theta - 4)\hat{i} + (4r\sin\theta\cos\theta\hat{k}) \cdot (-4\sin\theta\hat{i} + 4\cos\theta\hat{j} + 0\hat{k})\, d\theta$$

$$= \int_0^{2\pi} (-64\sin\theta\cos^2\theta - 16\sin^2\theta + 16\sin\theta + 192\sin\theta\cos^2\theta\, d\theta)$$

$$= 16\pi$$

$$\frac{\delta\Phi}{\delta r} = (\cos\theta, \sin\theta, 0)$$

$$\frac{\delta\Phi}{\delta\theta} = (r\sin\theta, r\cos\theta, 0)$$

$$\frac{\delta\Phi}{\delta r} \times \frac{\delta\Phi}{\delta\theta} = \hat{i}(0+0) - \hat{j}(0) + \hat{k}(r\cos^2\theta + r^2 s^2\theta)$$

$$= r\hat{k}$$

$$\iint (\text{curl } F) \cdot n\, dA$$

$$= \int_0^4 \int_0^{2\pi} (-2)(0)\hat{j} + (3r\sin\theta + 1)\hat{k}(r\hat{k})\, dr\, d\theta$$

$$\int_0^4 \int_0^{2\pi} (3r^2\sin\theta + r)\, dr\, d\theta$$

$$\int_0^4 (3r^2\cos\theta + r\theta)_0^{2\pi}\, d\theta$$

$$\int_0^4 -3r^2 + 2\pi r + 3r^2 - 0\, dr$$

$$= \left[-\frac{3r^3}{3} + \frac{2\pi r^2}{2} + \frac{3r^3}{3}\right]_0^4$$

$$= -64 + 16\pi + 64 = 16\pi \text{ Ans}$$

—x—